# Partial NOMA-Based Resource Allocation for Fairness in LTE-U System

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Abstract—In order to tackle the spectrum scarcity problem and enhance the spectrum efficiency, deploying LTE in unlicensed band (LTE-U) is an emerging technology for supporting massive connections in future networks. By taking into account of the coexistence between the LTE-U cellular user equipments (CUEs) and the legacy Wi-Fi stations (STAs) in the unlicensed band, a partial non-orthogonal multiple access (NOMA)-based scheme is proposed in this paper. By dividing all UEs into two groups and making the Wi-Fi STA as the UE with the weakest channel gain in its group, we can exploit the multiplexing gain of NOMA by introducing no extra modification to Wi-Fi STAs. Accordingly, a fairness-oriented resource allocation framework is formulated as a max-min problem to jointly optimize the inter-group time occupancy ratio and the intra-group power allocation when the guaranteed bit rate (GBR) requirements for each UE are considered. A modified two-dimensional bisection algorithm is proposed to search the optimal time occupancy ratio and the maxmin rate in this coexisting network. Numerical results validate the effectiveness of the partial NOMA scheme and outperform the traditional orthogonal multiple access method, in terms of both efficiency and robustness.

*Index Terms*—LTE in unlicensed spectrum, non-orthogonal multiple access, fairness, resource allocation.

# I. INTRODUCTION

Based on the latest forecast from the industry [1], the global IP data traffic will reach 396 exabytes per month in 2022, 71% of which are composed of the mobile data traffic. It is expected that the tremendous traffic demands can lead to a potential network congestion, which poses technical challenges to future mobile networks. To address this issue, deploying LTE-like networks in the unlicensed spectrum (known as LTE-U) is an emerging technology by augmenting the mobile cellular systems with the unlicensed spectrum [2]. However, the current unlicensed spectrum is almost occupied exclusively by the Wi-Fi devices working with IEEE 802.11 protocols. Basically, the Wi-Fi devices access the unlicensed channel via a distributed carrier sensing multiple access/collision avoidance (CSMA/CA) mechanism, while the LTE-U system inherently follows a centralized scheduling paradigm [3]. This difference raises considerable concern about the coexistence between the two networks [4].

There are many existing studies on the coexistence between LTE-U and Wi-Fi networks in terms of both fairness and

efficiency. In [5], Wang et al. analyze the coexistence performance via a stochastic geometry method and find out that the high density of LTE-U base stations (BSs) results in serious performance degradation for the Wi-Fi network, while Wi-Fi has much smaller effects on LTE-U. In [6], Cano et al. point out that the performance degradation in LTE-U and Wi-Fi coexisting networks is mainly caused by the heterogeneity of their channel access approaches and it could be alleviated by increasing the duration of the LTE-U's air time at the cost of the increased variability of delay for Wi-Fi transmissions. A resource allocation algorithm is proposed to balance the tradeoff between the overall network performance and the delay variance. In our previous work [7], considering the LTE-U BS with a listen-before-talk (LBT) scheme to access the unlicensed channel, we model the LTE-U/Wi-Fi interaction via two Markov chains, based on which a joint power and channel allocation algorithm is proposed for proportional fairness in radio resource allocation.

Most of the existing works about fair access focus on airtime-related allocation, i.e., making the Wi-Fi networks and the LTE-U networks access the channel in different time slots and then optimizing their time occupancy ratio. Such time division multiplexing mechanisms can avoid the internetwork interference and guarantee the fairness of average transmission rate for each network. But, the alternative access of the LTE-U network and Wi-Fi network can introduce extra delay to both networks, especially when just sending some small packets. On the other hand, the guard space between the two adjacent accesses leads to the unlicensed spectrum underused [8]. To overcome the limitations, non-orthogonal multiple access (NOMA) is considered in this paper to enable the concurrent downlink transmission of the LTE-U cellular user equipments (CUEs) and the Wi-Fi stations (STAs) in the same time-frequency resource block [9]. When the LTE-U BS and Wi-Fi access point is colocated and coordinated, by adopting successive interference cancellation (SIC) in NOMA, the downlink signals of the users with lower channel gains can be decoded by those with higher channel gains due to a higher received signal-to-interference-noise-ratio (SINR) [10]. Regardless of the power allocation scheme, there is no interference caused from UEs with lower channel gains to those with higher channel gains. Therefore, the NOMA can reach a larger achievable capacity region than that of the traditional orthogonal multiple access (OMA), such as time-division-multiple-access (TDMA) along with frequencydivision-multiple-access (FDMA), and can achieve a lower

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delay due to its concurrent transmissions [11].

However, due to the incapability of SIC in legacy Wi-Fi STAs, the direct employment of NOMA in the LTE-U and Wi-Fi coexisting network is challenging, even when there is a coordination between the two networks. In oder to avoid any extra modification to the inherent Wi-Fi STAs, we propose a partial NOMA-based scheme, based on which all LTE-U CUEs with lower channel gains than those of Wi-Fi STAs access the channel in different time slots, while the remaining CUEs and the ongoing-transmission Wi-Fi STA access the channel via a NOMA method. To our best knowledge, this is the first work studying how to integrate NOMA into the LTE-U and Wi-Fi coexisting networks. With the consideration of fairness in the coexisting networks, a max-min optimization problem is formulated for a joint power and time occupancy ratio allocation. A two-dimensional bisection algorithm with reduced complexity is then presented to search the max-min rate in the networks. Simulation results demonstrate that our proposed scheme outperforms the traditional TDMA scheme in terms of both efficiency and robustness.

The remainder of the paper is organized as follows. The system model under consideration is presented in Section II. In Section III, we formulate the joint power and time occupancy ratio allocation into a max-min problem and propose the twodimensional bisection algorithm. Performance evaluation for the proposed scheme is given in Section IV, followed by conclusions in Section V.

## II. SYSTEM MODEL

We consider that a hybrid access point (HAP) integrates both Wi-Fi and cellular components in one physical small base station (SBS), operating in an unlicensed channel [12]. The Wi-Fi and cellular systems are supported by the same service provider or can be coordinated by different service providers. The associated CUEs are indexed by  $n \in \mathcal{N} = \{1, 2, 3, ..., N\}$ . Because there is only one active transmission link at a time in a Wi-Fi local network based on the CSMA/CA mechanism, we consider only one Wi-Fi STA indexed by w in each resource allocation scenario. The whole UE set is denoted by  $\mathcal{U} = \mathcal{N} \cup$  $\{w\}$ . Only downlink transmission is considered in this paper and the HAP accesses the unlicensed channel via an LBT or carrier-sensing-adaptive-transmission mechanism. The uplink transmission of the CUEs can use the mechanism of [2] and the Wi-Fi STAs still follow the traditional IEEE 802.11. Each UE has a minimum guaranteed bit rate (GBR) QoS requirement  $\overline{R}_n, n \in \mathcal{U}$ . The channel power gains from the HAP to the CUEs are denoted by  $g = (g_1, g_2, ..., g_N)$ . Without loss of generality, it is assumed that the power gains are sorted in a descending order, i.e.,  $g_1 \ge g_2 \ge ... \ge g_N$ . The channel power gain of the Wi-Fi STA w is expressed by  $g_w$  and we consider  $g_w \leq g_m$  and  $g_w \geq g_{m+1}$ .<sup>1</sup> For presentation simplicity, we assign w a value such that m < w < m+1. The channel power gains of the CUEs can be obtained by the channel quality indicator reporting mechanism similar with the LTE system [13]. And the channel power gain of the Wi-Fi STA can be



Fig. 1. (a) Pure NOMA. (b) Partial NOMA.

obtained by the channel quality measurement mechanism of IEEE 802.11.

Aiming at maximizing the achievable capacity region of the UEs, we consider all the associated CUEs are capable of superposition decoding to perform SIC. Thus, the HAP can provide the downlink transmission via NOMA as shown in Fig. 1(a), i.e., all signals of the associated UEs can be modulated in the same time-frequency resource block, where a CUE with a higher channel power gain can decode the signals of the UEs with lower channel power gains. Therefore, given the power allocation  $\mathbf{P} = (P_n)_{n \in \mathcal{U}}$  of all UEs, the transmission rate of CUE *n* is given by

$$R_n = B \log \left( 1 + \frac{P_n g_n}{\sum\limits_{n' < n, n' \in \mathcal{U}} P_{n'} g_n + N_0} \right), \forall n \in \mathcal{N} \quad (1)$$

where *B* is the bandwidth of the unlicensed channel and  $N_0$  is the power of the received noise. However, the Wi-Fi STA is assumed to follow the physical-layer standard of IEEE 802.11 and cannot perform the SIC, regardless of whether it has better SINR than other CUEs. Therefore, the Wi-Fi STA has to treat all the signals of the CUEs as interference and its transmission rate is expressed by

$$R_w = B \log \left( 1 + \frac{P_w g_w}{\sum\limits_{n \in \mathcal{N}} P_n g_w + N_0} \right).$$
 (2)

The heterogeneity of the Wi-Fi STAs has a serious impact on the performance of the whole network, due to the mutual interference with those CUEs with low channel power gains.

To cancel such mutual interference, a partial NOMA-based mechanism is designed as shown in Fig. 1(b). We divide all UEs into two groups: 1) The Wi-Fi STA and all CUEs with a channel power gain higher than that of the Wi-Fi STA are assigned into the first group, with  $U_1 = \{1, 2, 3, ..., m, w\};$ 2) The rest CUEs are assigned into the second group, with  $\mathcal{U}_2 = \{m+1, m+2, \dots, N\}$ . The number of the UEs in group  $i \in \{1,2\}$  is denoted by  $U_i = |\mathcal{U}_i|$ , where  $|\cdot|$  denotes set cardinality. The two groups are assigned with different dedicated time resources and their time occupancy ratios are denoted by  $\alpha = (\alpha_1, \alpha_2)$  and  $\alpha_1 + \alpha_2 = 1$ . UEs in the same group access the channel with a NOMA-based mechanism, and different groups of UEs access the channel with a TDMAbased mechanism. The HAP has the same power limit  $\bar{P}$ for both groups' transmission. Based on such a grouping scheme, the Wi-Fi STA has the weakest channel power gain in the first group, and there is no performance degradation due to its incapability of superposition decoding. Therefore, the

<sup>&</sup>lt;sup>1</sup>In the case that the channel power gain of the Wi-Fi STA  $(g_w)$  is larger than  $g_1, m = 0$ ; and in the case that  $g_w < g_N, m = N$ .

transmission rate of UE n in group  $i \in \{1, 2\}$ ) is denoted as

$$R_n = \alpha_i B \log \left( 1 + \frac{P_n g_n}{\sum\limits_{n' \in \mathcal{U}_i, n' < n} P_{n'} g_n + N_0} \right), n \in \mathcal{U}_i.$$
(3)

# **III. PROBLEM FORMULATION**

In order to make all UEs be fairly served in the unlicensed channel, we aim at maximizing the minimal transmission rate of all UEs while guaranteeing their GBR requirements, i.e.,

$$\mathbf{P0}: \max_{\boldsymbol{P},\boldsymbol{\alpha}} \min_{n \in \mathcal{U}} R_n \tag{4a}$$

$$s.t.R_n \ge \overline{R}_n, \forall n \in \mathcal{U}$$
(4b)

$$\sum_{n \in \mathcal{U}_i} P_n \le \bar{P}, i = 1 \text{ or } 2 \tag{4c}$$

$$P_n \ge 0, \forall n \in \mathcal{U} \tag{4d}$$

$$\alpha_1 + \alpha_2 = 1, \alpha_1 > 0, \alpha_2 > 0 \tag{4e}$$

where  $R_n$  can be calculated by (3), (4c) and (4d) are constraints about the total transmission power limit, and (4e) is the constraint about the time occupancy ratio of the two groups. In this section, we first calculate the minimal power requirement for satisfying all the GBRs. Then, we determine the power allocation when the time occupancy ratio is given. After that, we present the searching algorithm for finding the optimal time occupancy ratio.

# A. Minimal Total Transmission Power

If the total power limit,  $\bar{P}$ , is low, it is possible that no available resource allocation scheme can meet all UEs' GBR requirements. Therefore, we firstly calculate the minimal required power budget  $\bar{P}$  by solving

$$\mathbf{P1}: \min_{\boldsymbol{\alpha}, \boldsymbol{P}} \bar{\boldsymbol{P}}$$
(5a)

It is found out that P1 is difficult to solve due to the nonconvex constraint in (4b). However, when the time occupancy parameter  $\alpha_1$  ( $\alpha_2$ ) is given, calculating the minimal required power budget in each group can be decoupled by solving P2 for  $i \in \{1, 2\}$  as

$$\mathbf{P2}: \min_{\boldsymbol{P}_i} \sum_{n \in \mathcal{U}_i} P_n \tag{6a}$$

s.t. 
$$g_n P_n \ge \left(2^{\frac{\overline{R}_n}{\alpha_i B}} - 1\right) \left(\sum_{n' < n, n \in \mathcal{U}_i} g_n P'_n + N_0\right), \forall n \in \mathcal{U}_i$$
(6b)

$$P_n \ge 0, \forall n \in \mathcal{U}_i \tag{6c}$$

where  $P_i = (P_n)_{n \in U_i}$  is the power vector for UEs in group *i*, and (6b) indicates the minimal transmit power for UE *n* to guarantee its GBR, which can be derived from (4b).

**Lemma 1.** The minimal transmit power budget of group *i* derived from P2, denoted by  $Q_i = \min \sum_{n \in U_i} P_n$ , is obtained when (6b) holds with equality.

*Proof.* Suppose that the optimal power allocation  $P_i^* = (P_n^*)_{n \in \mathcal{U}_i}$  of P1 has a strict inequality of (6b) to hold for

UE *n*, i.e., 
$$g_n P_n^* > \left(2^{\frac{\overline{R}_n}{\alpha_i B}} - 1\right) \left(\sum_{\substack{n' < n, n \in \mathcal{U}_i}} g_n P_{n'}^* + N_0\right).$$

We can decrease the transmit power of UE n to  $P'_n$  to make the equality hold while the power for other UEs remains the same. By constructing such a power allocation scheme, the transmission rates of all UEs can still satisfy the GBR because (6b) holds for each UE. However, the total power is less than the optimal power allocation, which conflicts with its minimization optimality and ends the proof.

Based on Lemma 1, we can obtain the minimal transmit power budget in group *i* with given  $\alpha_i$  by solving the following linear equations for  $P_i$ :

$$\mathbf{A}\left(\alpha_{i}, \overline{\mathbf{R}}_{i}\right) \mathbf{P}_{i}^{\mathrm{T}} = \mathbf{C}^{\mathrm{T}}\left(\alpha_{i}, \overline{\mathbf{R}}_{i}\right)$$
(7)

where  $\overline{\mathbf{R}}_i = (\overline{R}_n)_{n \in \mathcal{U}_i}$  is the GBR vector of the UEs in group i,  $\mathbf{A} (\alpha_i, \overline{\mathbf{R}}_i) = (\mathbf{a}_1^{\mathrm{T}}, \mathbf{a}_2^{\mathrm{T}}, ..., \mathbf{a}_{U_i}^{\mathrm{T}})^{\mathrm{T}}$  is a coefficient matrix of dimensions  $U_i \times U_i$ ,  $\mathbf{a}_n$   $(n \in \mathcal{U}_i)$  is the linear coefficient vector for constraint (6b) of UE n where all the first n-1 elements are  $\left(2^{\frac{\overline{R}n}{\alpha_i B}} - 1\right)g_n$ , the n-th element is  $-g_n$ , and all the rest are 0s, and  $C^{\mathrm{T}} (\alpha_i, \overline{\mathbf{R}_i})$  is a  $U_i \times 1$  constant vector with the n-th  $(n \in \mathcal{U}_i)$  element being  $-\left(2^{\frac{\overline{R}n}{\alpha_i B}} - 1\right)N_0$ . Therefore, given  $\alpha_i$ , the power allocation for the minimal power budget can be calculated as

$$\boldsymbol{P}_{i}^{\mathrm{T}} = \mathbf{A}^{-1} \left( \alpha_{i}, \overline{\boldsymbol{R}}_{i} \right) C^{\mathrm{T}} \left( \alpha_{i}, \overline{\boldsymbol{R}}_{i} \right)$$
(8)

and we obtain the minimal power budget in group i when given  $\alpha_i$  as  $Q_i = P_i \mathbf{1}^T$ , where **1** is a  $1 \times U_i$  vector with all elements being 1.

After obtaining the minimal power budget when given  $\alpha_1$  or  $\alpha_2$ , we can transform P1 to  $\min_{\alpha_1} \max \{Q_1(\alpha_1), Q_2(1-\alpha_1)\}$ . In addition, it is obvious that  $Q_1(Q_2)$  is a monotonic decreasing (increasing) function in terms of  $\alpha_1$  and the value of  $Q_1(Q_2)$  ranges from 0 to  $+\infty$ . Therefore, we use a simple bisection searching algorithm to find  $\alpha_1$  until  $Q_1(\alpha_1) = Q_2(1-\alpha_1)$ , to get the minimal power requirement  $\overline{P} = Q_1(\alpha_1) = Q_2(1-\alpha_1)$ .

# B. Optimal Power Allocation

The original problem, P0, is difficult to solve on account of non-convexity of the objective function. However, when  $\alpha_1$  or  $\alpha_2$  is fixed, we can formulate the max-min problem in each group for  $i \in \{1, 2\}$  as

**P3:** 
$$\max_{P_i} \min_{n \in \mathcal{U}_i} R_n$$
(9a)

s.t. 
$$R_n \ge \overline{R}_n, \forall n \in \mathcal{U}_i$$
 (9b)

$$\sum_{n \in \mathcal{U}} P_n \le \bar{P} \tag{9c}$$

$$P_n \ge 0, \forall n \in \mathcal{U}_i.$$
(9d)

**Lemma 2.** When feeding the optimal time occupancy ratios of P0,  $\alpha^*$ , into P3, the minimal rate of all UEs by solving P3 is the same as the max-min rate derived from P0.

*Proof.* We denote  $S_i^*$  as the minimal value of all UEs' transmission rates in group *i* by solving P0, and  $S_i^{\#}$  as that obtained from P3 when given  $\alpha^*$ . Then, we have  $S_i^{\#} \geq S_i^*$ for both i = 1 and 2 due to the optimality of P3. Thus, we obtain that  $\min(S_1^{\#}, S_2^{\#}) \geq \min(S_1^*, S_2^*)$ . In addition, the optimality of P0 leads to  $\min(S_1^*, S_2^*) \ge \min(S_1^\#, S_2^\#)$ , and thus  $\min(S_1^*, S_2^*) = \min(S_1^\#, S_2^\#)$ , which ends the proof.  $\Box$ 

Based on Lemma 2, once we know the optimal time occupancy ratio allocation, the optimal power allocation can be decoupled by maximizing the minimal transmission rate in each group, i.e., solving problem P3.

**Theorem 1.** Given  $\alpha$ , for the transmission rates of all UEs in group i under the optimal power allocation result derived from P3, denoted by  $R_n$ 's  $(n \in U_i)$ , and the corresponding minimal value of all UEs' transmission rates in group i, denoted by  $S_i^{\#}$ , we have

$$R_n = \begin{cases} \overline{R}_n, & \text{if } S_i^{\#} \leq \overline{R}_n \\ S_i^{\#} & \text{if } S_i^{\#} > \overline{R}_n \end{cases}, \quad \forall i \in \{1, 2\}, \ n \in \mathcal{U}_i.$$
(10)

*Proof.* Suppose that  $P_i^{\#} = (P_n^{\#})_{n \in \mathcal{U}_i}$  is the optimal power allocation of P3. Firstly, we assume some UE  $j \in \mathcal{U}_i$  conflicts against the first part of (10), i.e.,  $R_j > \overline{R}_j \ge S_i^{\#}$ . It is obvious that  $R_n \ge S_i^{\#}$  for  $n \in \mathcal{U}_i \setminus \{j\}$ . For this case, we make the allocated power for UE j from  $P_j^{\#}$  to  $P_j^{\#} - \delta$ , where  $\delta (> 0)$  is small enough to guarantee that the modified transmission rate  $P_i \ge \overline{R}_i \ge S_i^{\#}$ . rate  $R'_j > \overline{R}_j > S_i^{\#}$ . At the same time, the power of all the rest UEs is magnified by  $\eta = \frac{\overline{P} - P_j^{\#} + \delta}{\overline{P} - P_j^{\#}}$  times. Thus, given  $\eta > 1$ , the SINRs of all UEs except j become at least  $\frac{\eta P_n^{\#} g_n}{\eta I_n + N_0}$ , larger than the original SINR  $\frac{P_n^{\#}g_n}{I_n+N_0}$ . By constructing such a new power allocation, all UEs except j in group i has a larger transmission rate, i.e.,  $R'_n > R_n \ge S_i^{\#}$  while UE j still has a rate larger than  $S_i^{\#}$ . Therefore, we have  $\min_{n \in \mathcal{U}_i} R'_n > C_i \in \mathcal{U}_i$  $S_i^{\#}$  , conflicting with the optimality of  $S_i^{\#}.$  In addition, if some UE j conflicts against the second part of (10), we have  $R_j > S_i^{\#} > \overline{R}_j$ . Similarly, we decrease the power of UE j and allocate the power decrement to the rest UEs. Then, we can have a larger minimal rate than  $S_i^{\#}$ , conflicting with the optimality of  $S_i^{\#}$ , which ends the proof.

Based on Theorem 1, we develop a bisection searchingbased algorithm to obtain the max-min transmission rate in each group for  $i \in \{1, 2\}$  when given  $\alpha_1$  or  $\alpha_2$ . Details are given in Algorithm 1. We first initialize the estimated upper bound and lower bound of the max-min rate and the stopping criteria parameter  $\epsilon$ . Then, we iteratively update the lower and upper bounds. The basic idea is to first check whether the maximal GBR can be set as the minimal transmission rate of all UEs in this group. If not, we set the maximal GBR as the upper bound of the max-min rate of UEs in this group, and continue to check for a smaller GBR. Once we have a feasible power allocation, under which some UE's GBR can be set as the minimal transmission rate of all UEs in this group, this UE's GBR is set as the lower bound of the max-min rate. After we get the upper bound and lower bound of the maxmin rate, we further use a standard bisection method in the while-loop to get the optimal max-min rate  $S_i^{\#}$  with tolerance parameter  $\epsilon$ .

Algorithm 1 Bisection-based algorithm for the optimal power allocation in group i

# Initialize:

Set the upper bound of the max-min rate of group i as  $S_i^{\rm U} = \min_{n \in \mathcal{U}_i} \alpha_i \log \left(1 + \frac{g_n \bar{P}}{N_0}\right)$ , the lower bound as  $S_i^{\rm L} = \max_{n \in \mathcal{U}_i} \overline{R}_n$ , user set  $\mathcal{D} = \mathcal{U}_i$ , the stopping criteria parameter  $\epsilon$ ; for j = 1 to  $U_i$  do

Set the transmission rate vector  $\mathbf{R}_i = (R_n)_{n \in \mathcal{U}_i}$  with

the assumption of  $S_i^{\#} = S_i^{\text{L}}$  based on (10), i.e.,  $R_n = \begin{cases} \overline{R}_n, & \text{If } S_i^{\text{L}} \leq \overline{R}_n, \\ S_i^{\text{L}}, & \text{If } S_i^{\text{L}} > \overline{R}_n, \end{cases} \forall n \in \mathcal{U}_i;$ Solve the following equations about  $P_i$ 

$$\mathbf{A}(\alpha_i, \mathbf{R}_i) \mathbf{P}_i^{\mathrm{T}} = \mathbf{C}^{\mathrm{T}}(\alpha_i, \mathbf{R}_i); \qquad (11)$$

Flag  $\leftarrow$  (11) is solvable &  $P_i \succeq \mathbf{0}$  &  $P_i \mathbf{1}^{\mathrm{T}} \leq \overline{P}$ ; if Flag == 1 then Break; else Set  $S_i^{\mathrm{U}} \leftarrow S_i^{\mathrm{L}}$ ;  $\mathcal{D} \leftarrow \mathcal{D} \setminus \{ \arg \max_{n \in \mathcal{D}} \overline{R}_n \}$ ; Set  $S_i^{\mathrm{L}} \leftarrow \max_{n \in \mathcal{D}} \overline{R}_n$ ; end if if  $j == U_i$  & Flag == 0 then **Output**: Not a feasible  $\alpha_i$ ; end if end for while  $S_i^{U} - S_i^{L} > \epsilon$  do  $S_i^{M} \leftarrow (S_i^{L} + S_i^{U})/2;$ Set the transmission rate  $\mathbf{R}_i$  with the assumption of  $S_i^{\#} = S_i^{\mathrm{M}}$  based on (10); Solve the equations (11) and obtain  $P_i$  and the Flag; if Flag == 1 then  $\begin{array}{c} \text{In } \text$ end if end while **Output**  $P_i$  as the optimal power resource allocation.

# C. Optimal Time Occupancy Ratio

We have discussed how to determine the optimal power allocation for the UEs in each group, when the time occupancy ratio  $(\alpha_1, \alpha_2)$  is given. In this subsection, we further study how to assign the time-domain resources for the two UE groups.

With Lemma 2 and Algorithm 1, we can transform problem P0 to maximize  $\min \left\{ S_1^{\#}(\alpha_1), S_2^{\#}(1-\alpha_1) \right\}$ . It should be noted that the max-min rate in group  $i, S_i^{\#}$ , is a continuous increasing function of time occupancy ratio  $\alpha_i$ .

Theorem 2. Given the optimal time occupancy ratio of PO,  $\boldsymbol{\alpha}^* = (\alpha_1^*, \alpha_2^*)$ , the minimal transmission rate,  $S_i^{\#}$ , in group *i* obtained from Algorithm 1 follows

$$S_1^{\#} = S_2^{\#}, \text{ if } S_i^{\#} > \min_{n \in \mathcal{U}_i} \overline{R}_n, \forall i \in \{1, 2\}.$$
 (12)

*Proof.* If (12) does not hold, without loss of generality, we assume  $S_1^{\#} > S_2^{\#}$ , and thus the minimal value of all UEs' transmission rates in this case is  $S_2^{\#}$ . Because  $S_i^{\#}$  is a continuous increasing function of  $\alpha_i$ , there exists an  $\alpha'_1 < \alpha_1^*$  to make the max-min transmission rate in group 1 as  $\tilde{S}_1^{\#} = \left[S_1^{\#} + \max\left\{S_2^{\#}, \min_{n \in \mathcal{U}_1} \overline{R}_n\right\}\right]/2 > S_2^{\#}$ . Correspondingly, the time occupancy ratio of group 2 is  $\alpha'_2 = 1 - \alpha'_1 > \alpha_2^*$ , leading to  $\tilde{S}_2^{\#} > S_2^{\#}$ . Therefore, the minimal rate under the revised time occupancy ratio  $(\alpha'_1, \alpha'_2)$  is min  $\left\{\tilde{S}_1^{\#}, \tilde{S}_2^{\#}\right\} > S_2^{\#}$ , which conflicts with the optimality of  $\alpha^*$ .

Based on Theorem 2, we can develop a bisection algorithm for  $\alpha_1$  (or  $\alpha_2$ ) where, under each  $\alpha$ , the max-min rates pair,  $(S_1^{\#}, S_2^{\#})$ , is obtained based on Algorithm 1, and a larger occupancy ratio is gradually assigned to the group with a lower max-min rate.

#### D. Optimal Joint Power and Time Occupancy Ratio Allocation

We have two bisection algorithms for the power allocation and time occupancy ratio allocation, respectively. However, Algorithm 1 needs to be performed for each  $\alpha$  until the accurate power allocation is obtained, which is computational intensive and inefficient. In fact, we can compare the upper bounds and lower bounds to determine which group should be assigned a higher time occupancy ratio. Then, a joint power and time occupancy ratio allocation algorithm is developed in Algorithm 2, with a reduced complexity.

# Algorithm 2 Joint power and time occupancy ratio allocation Initialize:

Set the lower and upper bounds for  $\alpha_1$  as  $\alpha_1^{\rm L} = 0$  and  $\alpha_1^{\rm U} = 1$ ; Set the initial time occupancy ratio as  $\alpha_1 = \alpha_2 = 0.5$ ; Set the upper bound of the max-min rate in both groups as  $S_i^{\rm U} = \min_{n \in \mathcal{U}_i} \alpha_i \log \left(1 + \frac{h_n \bar{P}}{N_0}\right)$ ; the lower bound as  $S_i^{\rm L} = \max_{n \in \mathcal{U}_i} \overline{R}_n$ , and the stopping criteria parameter as  $\epsilon$ ; while  $\alpha_1^{\rm U} - \alpha_2^{\rm L} > \epsilon \parallel \max_i \left(S_i^{\rm U} - S_i^{\rm L}\right) > \epsilon$  do for  $i \in \{1, 2\}$  do Perform Algorithm 1 until the upper bound or lower bound of group i has an update in the While-loop;

end for if  $S_1^{U} < S_2^{L}$  then Set  $\alpha_1^{L} \leftarrow \alpha_1$ ,  $\alpha_1 \leftarrow (\alpha_1^{L} + \alpha_1^{U})/2$ ,  $\alpha_2 \leftarrow 1 - \alpha_1$ ; Reinitialize Algorithm 1 for both groups; else if  $S_1^{L} > S_2^{U}$  then Set  $\alpha_1^{U} \leftarrow \alpha_1$ ,  $\alpha_1 \leftarrow (\alpha_1^{L} + \alpha_1^{U})/2$ ,  $\alpha_2 \leftarrow 1 - \alpha_1$ ; Reinitialize Algorithm 1 for both groups; end if end while Output  $\alpha$  and  $(P_i)_{i \in \{1,2\}}$  from Algorithm 1.

In Algorithm 2, if we find out that the lower bound of the max-min rate in group 1 is larger than the upper bound of that in group 2, indicating that group 1 has been assigned a large time occupancy ratio, we need to lower the upper bound of  $\alpha_1$ , and vice versa. With the proposed algorithm, the time occupancy ratio can be adjusted even when there is



Fig. 2. Comparison of GBR guarantee ratio and the minimal transmission rate.

no accurate value of the max-min rate in each group. The concurrent bisections on both the power allocation and the time occupancy ratio allocation can significantly reduce the unnecessary calculations of the explicit max-min rate in each group, especially when there is a serious mismatch between these two groups.

#### **IV. PERFORMANCE EVALUATION**

In this section, simulation results are presented to evaluate the performance of the proposed joint power and time occupancy ratio allocation algorithm. We consider a HAP operates on a single channel of 20 MHz bandwidth in 5 GHz unlicensed frequency bands and is located at the centre of a square area with size 500m  $\times$  500m. Unless otherwise specified, the maximum transmit power of this HAP is set as 1 Watt. The received noise power density is set as -174 dBm/Hz. One Wi-Fi STA and 5 CUEs are randomly located in the coverage area. Isotropic antennas are installed at the HAP, the Wi-Fi STA, and all CUEs. Both free space path loss and Rayleigh channel fading are considered for a downlink channel from the HAP to a CUE (or the Wi-Fi STA), i.e.,  $g_n = \left(\frac{\lambda}{4\pi d_n}\right)^2 v_n$ , where  $\lambda$  is the wavelength,  $d_n$  is the distance (in meter) from the HAP to UE  $n, v_n$  is a random variable following an exponential distribution with parameter 1, representing the Rayleigh fading of the channel. The GBRs of all UEs in this scenario follow a uniform distribution over the range [2, 7] Mbps. The stopping criterion is set as  $\epsilon = 10^{-8}$ .

We select an ideal TDMA-based algorithm as the benchmark, because we consider that the TDMA provides an upper bound of the performance in traditional time-orthogonal access scheme (like LBT) for the LTE-U and Wi-Fi coexisting networks due to the lack of the time-domain guard space. The downlink power limit of the TDMA-based benchmark is set the same as that of NOMA. It first allocates sufficient time resources for each UE to satisfy its GBR requirement, and then gradually allocates the remaining resources to the UE with the least transmission rate. In the absence of the GBR requirement, all UEs have the same transmission rate under the TDMA-based algorithm.

In Fig. 2, we compare both the GBR guarantee ratio and the average minimal transmission rate of the proposed partial NOMA scheme and the TDMA scheme based on 10000



Fig. 3. Iteration process of Algorithm 2.



Fig. 4. Comparison between max-min rate and GBR.

experiments. When the transmit power of the HAP is not high enough or the channel quality of some UE is poor, it is difficult for the HAP to satisfy the GBR requirements of all UEs. However, it is noted that the guarantee ratio of NOMA is always larger than that of the TDMA scheme. This advantage is because NOMA can better exploit the heterogeneous GBR requirements and enhance the spectrum efficiency by enabling multiple UEs in the same time-frequency resource block.

Figure 3 shows the convergence process of the upper/lower bound of the max-min rate of group 1 and the upper/lower bound of the time occupancy ratio  $\alpha_1$ . We can see that  $\alpha_1$ evolves more smoothly, while the upper bound and the lower bound of max-min rate fluctuate. Each time  $\alpha_1$  has an update, the upper and lower bounds of the max-min rate need to be reset, as indicated in Algorithm 2, resulting in the fluctuations. After about 30 iterations, the time occupancy ratios achieve a rough convergence. Because our stopping criterion is small, i.e.,  $10^{-8}$ , the whole algorithm converges after about 430 iterations.

Figure 4 shows an example of the final transmission rates for each UE under both the TDMA scheme and the partial NOMA scheme, along with the GBR requirements. It is shown that both TDMA scheme and partial NOMA scheme aim to provide all UEs with the same transmission rate as indicated by the two lines in Fig. 4, after satisfying all UEs' GBR requirements. However, the achievable capacity region of this network is significantly expanded by the partial NOMA's multiplexing gain in the power domain and makes its max-min rate higher than that of TDMA.

## V. CONCLUSION

In this paper, we propose a novel partial NOMA architecture for the resource allocation in the LTE-U and Wi-Fi STA coexisting networks. Without making any additional modification to the Wi-Fi STAs, all UEs can benefit from the multiplexing gain of NOMA by the appropriately designed grouping scheme. In oder to exploit both the fairness and the efficiency in the network, a max-min-based optimization problem of all UEs' transmission rate is formulated for the joint power and time occupancy ratio allocation. Although the problem is non-convex, it is found out that the optimal max-min rate is achieved in the coexisting network when trying to make all UEs have the same throughput. Thus, a two-dimensional bisection algorithm is proposed and its optimality is proved. The simulation results validate the improved performance of the proposed scheme, over that of the traditional TDMA-based scheme.

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